representations z: 1) $z = G_*$; 2) $z = H_0G_*$; 3) $z = H_0G_*V_*\alpha_0^*$; 4) $z = H_0G_*V_*$; 5) $z = G_*V_*\alpha_0^*$. The values of the respective quantities at $\alpha = 0.02$ are taken as unity.

Topographic lines of equal values of the functional for $z = G_*V_*$ in the coordinates $\xi_{CO_2} - \xi_{N_2}$ are presented in Fig. 6. The value of P_z at the optimum point is taken as unity. Such pictures give a concept of the functional in the vicinity of the optimum point.

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CALCULATIONS OF ENERGY CHARACTERISTICS OF MULTICOMPONENT WORKING MEDIA IN CO_2 GASDYNAMIC LASERS BASED ON COMBUSTION PRODUCTS

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The working media of CO_2 gasdynamic lasers (GDL) operating on the combustion products of fuels containing the elementary composition C, H, O, N are, as a rule, multicomponent media. At stagnation temperatures $T_0 < 2000°K$ there are mainly CO, O_2, and H_2 present in them along with the main components CO_2, N_2, and H_2O. Before entering the nozzle the multicomponent media are in a state of total thermodynamic equilibrium. This makes it possible to use the thermodynamic approach developed earlier [1], based on the fact that a complex medium is characterized by the elemental composition and the stagnation temperature and pressure (T_0, p_0), for the analysis of their laser properties. Calculations of the amplification factors of such media and the corresponding analysis are presented in [2, 3]. The useful radiant energy $\bar{w}$ which can be obtained from a unit mass of the working medium is investigated in the present report.

Reports on calculations of the radiant energy and power in GDL can be divided arbitrarily into three groups. Estimating reports, which do not consider the generation process at all, belong to the first group. These are simple calculations either of the maximum energy attainable for extraction [4] or of an estimate of the generation power [5, 6]. The second group of reports includes more realistic calculations [7-12]; processes of generation and vibrational kinetics in the resonator are now considered in them, but their influence on the gasdynamic parameters of the stream are neglected. The stream in the resonator is assumed to be isothermal and to have a constant velocity.

Since the assumption of constancy of the gasdynamic parameters in the resonator is not always justified, the further improvement of the calculations is connected with allowance for the mutual influence of the vibrational kinetics, emission, and gasdynamics of the stream. This is the third group of reports [13-15]. In

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such an approach the problem of calculating $\bar{w}$ even for the simplest Fabry–Perot resonator is only solved numerically. With allowance for the fact that each separate system also requires the choice of the optimum coefficient of transmission t of the exit mirror, it becomes clear that the calculation of $\bar{w}$ for a large number of complex compositions requires considerable expenditures of computer time. Therefore, in the present report we chose a compromise approach consisting in the fact that the main calculations are made by an approximate method in which the mutual influence of the vibrational kinetics, emission, and gasdynamics of the stream is not taken into account. Further, in order to determine the limits of applicability of such an approach we made "exact" calculations of the specific power output which allow for the mutual influence of the vibrational kinetics, emission, and gasdynamics for several strongly differing compositions.

1. The assumption of constancy of all the gasdynamic parameters of the stream in the resonator simplifies the problem of determining $\bar{w}$ with allowance for losses and the saturation effect. The corresponding analytical expressions are given in [12, 15] for the case when the symmetric types of CO_2 vibrations are in equilibrium with the translational degrees of freedom ($T_1 = T_2 = T_{1,2} = T$).† With a number of additional assumptions the value of the specific emission energy is calculated especially simply for a Fabry–Perot resonator of the maximum length (along the stream) with the optimum coefficient of transmission of the exit mirror [15],

$$\bar{w} = \bar{w}_{\max}\eta_r; \tag{1.1}$$

$$\bar{w}_{\max} = \eta_q \bar{\varepsilon}_4^0, \quad \eta_r = \left[1 - \sqrt{(\alpha + \beta L)/k_0 L}\right]^2, \tag{1.2}$$

where $\bar{w}_{\max}$ is the maximum specific energy available for extraction in kilojoules per kilogram; η_r is the efficiency of the resonator; η_q is the quantum efficiency of the CO_2 laser; $\bar{\varepsilon}_4^0$ is the specific vibrational energy of N_2 at the entrance to the resonator; α is the coefficient of absorption of the mirror; β is the coefficient of linear attenuation of the radiation; k_0 is the unsaturated amplification factor at the entrance to the resonator; L is the distance between the mirrors.

By assigning L and the value of the losses $\alpha + \beta L$ and using the results of a numerical calculation of the flow of the working medium through the nozzle, one can approximately calculate $\bar{w}$ from Eqs. (1.1) and (1.2). The fact that $\bar{w}_{\max}$ in (1.2) depends only on $\bar{\varepsilon}_4^0$ is due to the neglect of the contribution of the vibrational energy of CO_2 to the generation. Moreover, the vibrational energy removed from the resonator by the stream was not allowed for in the derivation of (1.2). To allow for this, $\bar{w}_{\max}$ is calculated by analogy with [4]. The only difference is this: At the entrance to the resonator $T_3 \neq T_4$; we assumed the constancy of $T_{1,2}$ but not of T during flow in the resonator;‡ in calculating T_3' (here and later a prime refers to parameters at the exit from the resonator) it was assumed, from the condition that the amplification factor is reduced to zero, that the generation takes place on the transition P 20 ($00^01 \rightarrow 10^00$). The values of the vibrational temperatures of N_2 and CO_2 at the entrance to the resonator (T_4^0, T_3^0, $T_{1,2}^0$) were taken from numerical calculations of the flow of multicomponent media through nozzles [2, 3, 16]. With allowance for the fact that $T' = T_{1,2}^0$ and $T_4' = T_3'$, with good accuracy, we have

$$\bar{w}_{\max} = (\bar{N}^0 - \bar{N}')\hbar\omega,$$

$$\bar{N}^0 = \left[n_{CO_2}\left(1 - e^{-\Theta_3/T_3^0}\right)\left(1 - e^{-\Theta_2/T_{1,2}^0}\right)^2\left(1 - e^{-\Theta_1/T_{1,2}^0}\right)e^{-\Theta_3/T_3^0} + n_{N_2}\left(1 - e^{-\Theta_4/T_4^0}\right)e^{-\Theta_4/T_4^0}\right]N_A, \tag{1.3}$$

$$\bar{N}' = \left[n_{CO_2}\left(1 - e^{-\Theta_3/T_3'}\right)\left(1 - e^{-\Theta_2/T_{1,2}^0}\right)^2\left(1 - e^{-\Theta_1/T_{1,2}^0}\right)e^{-\Theta_3/T_3'} + n_{N_2}\left(1 - e^{-\Theta_4/T_3'}\right)e^{-\Theta_4/T_3'}\right]N_A,$$

where n_{CO_2} and n_{N_2} are the numbers of moles of CO_2 and N_2, respectively, per unit mass of the working medium; Θ_i is the characteristic vibrational temperature of the i-th mode; N_A is Avogadro's number; ω is the frequency of the laser transition P 20($00^01 \rightarrow 10^00$).

In order to clarify the dependence of the energy characteristics of multicomponent working media on their composition, we made calculations for 64 compositions having a fixed molar fraction of 0.6 for the element N. For this we used the results, obtained in [3], of numerical calculations of the flow of multicomponent media through a plane profiled nozzle of the minimum length with a throat height $h_* = 2 \cdot 10^{-4}$ m and a degree of nozzle expansion $A/A_* = 30$ at $T_0 = 1600°K$ and $p_0 = 20$ atm. The nozzle parameters were chosen so that the gas temperature at the nozzle cut was sufficiently low (~300°K) and the output $p_0 h_*$ was less than 1 atm·

†The indices 1, 2, 3, and 4 refer everywhere to the symmetric, deformation, and asymmetric modes of CO_2 and the vibrational mode of N_2, respectively.

‡"Exact" calculations (Sec. 2) showed that this assumption is justified. During the flow of working media through the resonator $T_{1,2}$ varies more weakly than T and this variation is small.

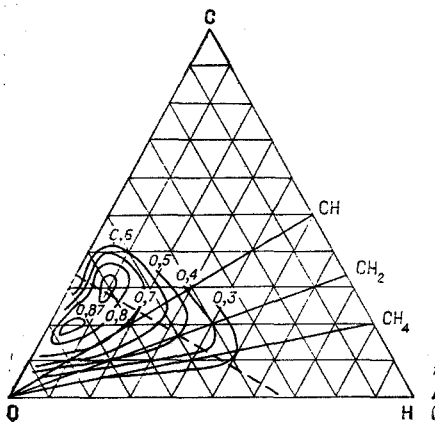

Fig. 1

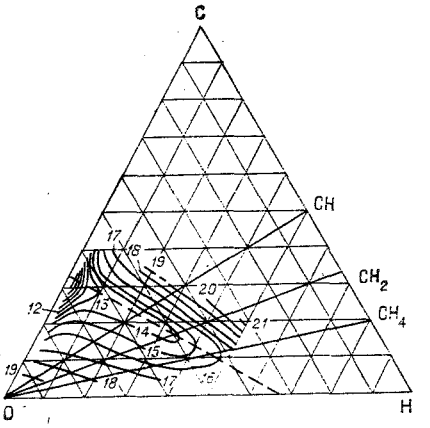

Fig. 2

cm (4). The quantities k_0, $\bar{w}$, and $\bar{w}_{max}$ are represented in the form of lines of equal values on triangular C, H, O diagrams of the elemental compositions.

The amplification factors k_0 (m^{-1}) of multicomponent media are presented in Fig. 1. It is seen that the region of elemental compositions characterized by high k_0 is split and the maxima are located on different sides of the triple-mixture line (the dashed line). The reasons for this phenomenon have been discussed earlier [3].

Lines of equal values of $\bar{w}_{max}$, calculated from Eqs. (1.3), are presented in Fig. 2. High values of $\bar{w}_{max}$ fall in those regions of elemental compositions which correspond to compositions with a low CO_2 content (the connection between the elemental and component compositions is given in [1]). This is connected with the fact that N_2 is the main carrier of vibrational energy, which changes into laser radiation energy and nitrogen loses vibrational energy mainly through a fast V–V exchange with CO_2 whose rate is proportional to the carbon dioxide concentration. It is seen that for these compositions the losses of N_2 vibrational energy through direct deactivation on H_2O molecules are low even at a high H_2O concentration. It should be noted that k_0 and $\bar{w}_{max}$ are important characteristics of working media, since they do not depend on the parameters of the resonator and allow one to estimate the useful specific power output.

Lines of equal values of the specific power output $\bar{w}$ for multicomponent media are presented in Fig. 3. Here the resonator efficiency (1.2) was calculated for total emission losses per passage ($\alpha + \beta L$) of 0.06 at a resonator width across the stream L = 2 m [17]. It is seen that high values of the specific, useful, emission energy can be obtained from those media which provide a sufficiently high stored energy $\bar{w}_{max}$ during expansion in the nozzle and an amplification considerably exceeding the losses.

2. The calculations of the specific power output $\bar{w}$ with allowance for the interaction of the vibrational kinetics, emission, and gasdynamics of the stream were made within the framework of the approximation of geometrical optics as in [14] (the errors made in [14] were corrected in accordance with [15]). We used the model of the flow of multicomponent media and the kinetic constants from [2, 3, 16].

The results of calculations of the specific power output made by such a method for four compositions are compared with approximate calculations of $\bar{w}$ (Sec. 1) in Table 1. The chosen compositions differ considerably from each other. Thus, a transition from composition 1, %: N_2 69.0; CO_2 27.6; H_2O 2.3; O_2 1.1 (all in percent by volume) to compositions 2, %: N_2 64.5; CO_2 12.9; H_2O 2.2; O_2 20.4; 3, %; N_2 65.4; CO_2 9.3; H_2O 3.8; CO 21.2; H_2 0.3; and 4, %: N_2 69.7; CO_2 9.3; H_2O 20.9; O_2 0.1 is characterized by a decrease in the CO_2 concentration in the working medium and a corresponding increase in the O_2, CO, and H_2O concentrations. The N_2 concentration varies insignificantly in this case. The elemental compositions corresponding to these compositions are marked by dots in Fig. 3. The optimum coefficients of transmission t_{opt} of the exit mirror, chosen in "exact" calculations, are also given in Table 1.

It is seen from Table 1 that the approximate calculations of $\bar{w}$ agree well with the "exact" calculations. In the most interesting region of compositions, when the molar fraction of the element is $H \leq 0.06$, the difference does not exceed 20%.

For working media with a high water content the approximate calculations give overstated values of $\bar{w}$. To find out the reasons for this we analyzed the assumptions made in the derivation of (1.1) and (1.2). The

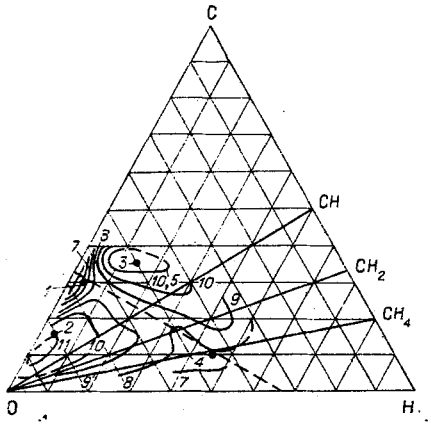

Fig. 3

TABLE 1

Composition number	Approximate calculation $\bar{w}$, kJ/kg	"Exact" calculation $\bar{w}$, kJ/kg	t_{opt}	Approximate calculation $\bar{w}$, kJ/kg	"Exact" calculation $\bar{w}$, kJ/kg	t_{opt}
1	6,7	6,5	0,24	7,8	7,6	0,27
2	11,0	11,4	0,29	12,8	13,2	0,27
3	10,8	9,1	0,24	12,8	11,2	0,22
4	7,9	4,9	0,19	9,9	6,4	0,17
$\alpha+\beta L$		0,06		0,03		

most important of them are: a) neglect of V–T deactivation of N_2; b) the approximate equality of T_3^0 and T_4^0 at the entrance to the resonator; c) the assumption of a very rapid suply of vibrational energy from N_2 molecules to the upper laser level, which is expressed by the limiting relation

$$\frac{Y_{N_2}k_{3,4}}{k_{3,2}^0}\left(\frac{2Lk_0}{2\alpha+2\beta+t}-1\right)\gg 1,$$

where Y_{N_2} is the molar fraction of nitrogen; $k_{3,4}$ is the rate constant of V–V exchange between the upper laser level and N_2; $k_{3,2}^0$ is the effective rate constant of the deactivation of the upper laser level.

The "exact" calculations showed that the incorporation of V–T channels of N_2 deactivation in the resonator leads to an increase of less than 10% in $\bar{w}$ even for composition 4. Consequently, the assumption a) is fully justified and cannot lead to the above-indicated overstatement of the results of the approximate calculations of $\bar{w}$. At sufficiently high H_2O concentrations $k_{3,2}^0$ increases. This leads to violation of the conditions b) and c). An analysis showed that nonfulfillment of condition b) can only lower the results of the approximate calculations. Hence, the observed overstatement of the results of the approximate calculations in comparison with those of the "exact" ones is due mainly to the violation of the assumption c).

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VOLTAGE–CURRENT CHARACTERISTIC OF GAS DISCHARGE WITH EXTERNAL IONIZATION

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Steady-state solutions of the discharge equations were obtained in [1] for a range of values of the external ionization source strength Q, discharge current j, and voltage U such that there was no impact ionization in the positive column, and the voltage was below the breakdown value.

The present paper treats a broader range of variation of Q, j, and U; inparticular, currents are considered for which impact ionization in the positive column is important, and the voltage is above the breakdown value.

Gas discharge with external ionization can be described by the following equations [1, 2]:

$$\frac{\partial q_-}{\partial t}+\frac{\partial j_-}{\partial x}=\alpha j_-+Q-\beta q_- q_+,\quad \frac{\partial q_+}{\partial t}-\frac{\partial j_+}{\partial x}=\alpha j_-+Q-\beta q_- q_+,$$
$$\frac{\partial E}{\partial x}=\frac{1}{\varepsilon}(q_--q_+),\quad j_-(0)=\gamma j_+(0),\quad j_+(d)=0,\quad \int_0^d E\,dx=U, \tag{1}$$

where q_- and q_+ are the electron and ion charge densities, j_- and j_+ are the electron and ion current densities, α is the impact ionization coefficient, β is the recombination coefficient, and γ is the coefficient of secondary emission at the cathode resulting from ion impact.

We use the same parameters for nitrogen as in [1]. The pressure is assumed atmospheric, and $\gamma = 0.01$. The method of finding numerical steady-state and transient solutions of system (1) is described in [1].

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